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Confining strings and RG flow in the (2+1)-dimensional Georgi-Glashow model and its SU(3)-generalization

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Abstract: Confining strings and RG flow at finite temperature are investigated in the (2+1)-dimensional Georgi-Glashow model. This is done in the limit when the electric coupling constant is much larger than the square root of mass of the Higgs field, but much smaller than the vacuum expectation value of this field. The modification of the Debye mass of the dual photon with respect to the case when it is considered to be negligibly small compared to the Higgs mass, is found. Analogous modifications of the potential of monopole densities and string coupling constants are found. At finite temperature, the mass of the Higgs field scales according to a novel RG equation. It is checked that in the limit when the original theory is reduced by the RG flow to the 2D XY model, the so-evolved Higgs mass is still much smaller than the squared electric coupling constant. The SU(3)-theory of confining strings is also discussed within the same approximations.

Keywords: Field Theories in Lower Dimensions, Nonperturbative Effects, Solitons Monopoles and Instantons, Confinement.

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Contents

1. Introduction

(2+1)-dimensional Georgi-Glashow model is known to possess confining properties at any values of electric coupling due to the presence of magnetic monopoles [1]. In the limit of infinitely heavy Higgs field, string representation of the Wilson loop in this model is therefore the same as in compact QED, which has been constructed in ref. [2]. One of the aims of the present paper (which will be realized in section 2) is to generalize the results of ref. [2] to the case when the mass of the Higgs field is large, compared to the Debye mass of the dual photon, but nevertheless finite. In this case, the respective dual field theory describing the grand canonical ensemble of monopoles has been obtained in ref. [3], and we shall use it as a starting point in our analysis. Another aim of the present paper (which will be realized in section 3) is to investigate the critical properties of the model under study at finite temperature by means of the RG flow equations. *A priori*, it is clear that the mass of the Higgs field should obey its own RG flow equation, which will be derived and analysed. After that, in section 4, the obtained results, regarding confining strings at the finite Higgs mass, will be generalized to the case of the SU(3) Georgi-Glashow model. Finally, the main results of the paper will be summarized in the Conclusions.

2. Monopole potential and confining strings

The euclidean action of the (2+1)-dimensional Georgi-Glashow model has the form [1]

$$S = \int d^3x \left[\frac{1}{4g^2}\left(F_{\mu\nu}^a\right)^2 + \frac{1}{2}\left(D_\mu\Phi^a\right)^2 + \frac{\lambda}{4}\left(\left(\Phi^a\right)^2 - \eta^2\right)^2\right], \tag{2.1}$$

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where the Higgs field Φ^a transforms by the adjoint representation, and $D_\mu \Phi^a \equiv \partial_\mu \Phi^a + \varepsilon^{abc} A_\mu^b \Phi^c$. In the one-loop approximation, the partition function of this theory reads [3]

$$\mathcal{Z} = 1 + \sum_{N=1}^{\infty} \frac{\zeta^N}{N!} \left[\prod_{i=1}^{N} \int d^3 z_i \sum_{q_i=\pm 1} \right] \times$$
$$\times \exp \left\{ -\frac{g_m^2}{2} \left[\int d^3 x d^3 y \rho_{\text{gas}}(\mathbf{x}) D_0(\mathbf{x}-\mathbf{y}) \rho_{\text{gas}}(\mathbf{y}) - \sum_{\substack{a,b=1 \\ a \neq b}}^{N} D_m(\mathbf{z}_a - \mathbf{z}_b) \right] \right\}. \quad (2.2)$$

Here, g_m is the magnetic coupling constant of dimensionality $[\text{length}]^{1/2}$ related to the electric one g according to the equation $g g_m = 4\pi$, $\rho_{\text{gas}}(\mathbf{x}) = \sum_{a=1}^{N} q_a \delta(\mathbf{x} - \mathbf{z}_a)$ is the density of monopole gas with q_a's standing for the monopole charges in the units of g_m. Next, in eq. (2.2), $m = \eta\sqrt{2\lambda}$ is the mass of the Higgs boson and

$$\zeta = \frac{m_W^{7/2}}{g} \delta\left(\frac{\lambda}{g^2}\right) \mathrm{e}^{-(4\pi/g^2) m_W \epsilon(\lambda/g^2)} \quad (2.3)$$

is the statistical weight of a single monopole (else called fugacity) with $m_W = g\eta$ being the mass of the W-boson. Here, ϵ is a slowly varying function equal to unity at the origin (i.e. in the Bogomolny-Prasad-Sommerfield (BPS) limit) [4] and $1.787\ldots$ at infinity [5], whereas the function δ is determined by the loop corrections. Finally, in eq. (2.2), $D_0(\mathbf{x}) \equiv 1/(4\pi|\mathbf{x}|)$ is the Coulomb propagator, and $D_m(\mathbf{x}) \equiv \mathrm{e}^{-m|\mathbf{x}|}/(4\pi|\mathbf{x}|)$ is the propagator of the Higgs boson.

The effective field theory describing the grand canonical partition function (2.2) can easily be obtained and reads [3]

$$\mathcal{Z} = \int \mathcal{D}\chi \mathcal{D}\psi \exp\left\{ -\int d^3 x \left[\frac{1}{2}(\nabla\chi)^2 + \frac{1}{2}(\nabla\psi)^2 + \frac{m^2}{2}\psi^2 - 2\zeta \mathrm{e}^{g_m \psi} \cos(g_m \chi) \right] \right\}, \quad (2.4)$$

where χ is the dual photon field, whereas the field ψ is an additional one. The latter field can be integrated out in the limit

$$g \gg \sqrt{m} \quad (2.5)$$

when the exponent in the last term on the r.h.s. of eq. (2.4) can be approximated by the terms not higher than the linear one. Let us clarify the latter statement. It is based on an obvious argument that the configuration of the ψ-field dominating in the partition function (2.4) is the one, at which every term of the action is of the order of unity. Then, demanding this for the kinetic term, $\int d^3x (\nabla\psi)^2 \sim 1$, we see that the characteristic wavelength L of the field ψ is related to the amplitude of this field as $L \sim |\psi|^{-2}$. Substituting this estimate into the second condition

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$\int d^3x m^2\psi^2 \sim L^3 m^2|\psi|^2 \sim 1$, we get $|\psi| \sim m^{1/2}$. The exponent of the ψ-field can be treated in the linear approximation when $g_m|\psi| \ll 1$, which upon the substitution of the obtained estimate for $|\psi|$ yields the above criterion (2.5).

In such a limit, gaussian integration over the field ψ yields[1]

$$\mathcal{Z} = \int \mathcal{D}\chi \exp\left\{-\int d^3x \left[\frac{1}{2}(\nabla\chi)^2 - 2\zeta\cos(g_m\chi)\right] + \right. \tag{2.6}$$
$$\left. + 2(g_m\zeta)^2 \int d^3x d^3y \cos(g_m\chi(\mathbf{x}))D_m(\mathbf{x}-\mathbf{y})\cos(g_m\chi(\mathbf{y}))\right\}.$$

Clearly, the last term here represents the correction to the standard result [1]. It stems from the fact that the mass of the Higgs field was considered to be not infinitely large compared to the standard Debye mass of the dual photon, $m_D = g_m\sqrt{2\zeta}$. The respective correction to m_D is positive, and the square of the full mass reads:

$$M^2 = m_D^2\left(1 + \frac{m_D^2}{m^2}\right). \tag{2.7}$$

Clearly, this result is valid at $m_D \ll m$ and reproduces m_D^2 in the limit $m \to \infty$. However, since we suppose now that m is not infinitely large, but is rather bounded from above by g^2, it looks reasonable to seek for the leading-order corrections in m_D/m to various quantities.

Another relation between the dimensionful parameters in the model (2.1), we shall adapt for the following analysis, is

$$g \ll \eta. \tag{2.8}$$

Clearly, this inequality parallels the requirement that η should be large enough to ensure the spontaneous symmetry breaking from SU(2) to U(1). In particular, from the inequalities (2.5) and (2.8) we immediately obtain:

$$\frac{\lambda}{g^2} \sim \left(\frac{m}{m_W}\right)^2 \ll \left(\frac{g}{\eta}\right)^2 \ll 1. \tag{2.9}$$

This means that we are working in the regime of the Georgi-Glashow model close to the BPS limit [4].

Note further that in the limit (2.5), the dilute gas approximation holds perfectly. Indeed, this approximation implies that the mean distance between monopoles, equal to $\zeta^{-1/3}$, should be much larger than the inverse mass of the W-boson. By virtue of eq. (2.3) and the fact that the function ϵ is of the order of unity, we obtain that this

[1] From now on, we omit the inessential normalization factors, implying that those are included in the respective integration measures.

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requirement is equivalent to the following one:

$$\sqrt{\frac{\eta}{g}}\delta\left(\frac{\lambda}{g^2}\right)\mathrm{e}^{-4\pi\eta/g}\ll 1\,. \tag{2.10}$$

It is known [6] that at $\lambda \ll g^2$ (cf. eq. (2.9)), the function δ diverges. However, it diverges in such a way that at $g \ll \eta$, the growth of δ does not spoil the exponential smallness of the l.h.s. of eq. (2.10) [7]. Another consequence of this fact is that in the limit (2.5), (2.8) of the Georgi-Glashow model, the Debye mass of the dual photon, m_D, remains exponentially small. In particular, the inequality $m_D \ll m$, under which the full mass (2.7) was derived, holds due to this smallness. Also, due to the same reason, the mean field approximation, under which the effective field theory (2.4) is applicable, remains valid as well with the exponential accuracy. Indeed, this approximation implies that in the Debye volume[2] m_D^{-3} there should contain many particles [1]. Since the average density of monopoles is equal to 2ζ (which can be seen either by calculating it directly according to the formula $V^{-1}\partial\ln\mathcal{Z}/\partial\ln\zeta$, applied to eq. (2.6) at $m_D \ll m$, or from the remark following after eq. (2.14) below), we arrive at the requirement $\zeta m_D^{-3} \gg 1$. Substituting the above-obtained value for m_D, we see that the criterion of applicability of the mean field approximation reads $g^3 \gg \sqrt{\zeta}$. Owing to the above-discussed exponential smallness of ζ, this inequality is satisfied.

Let us now derive the potential of monopole densities corresponding to the partition function (2.6). This can be done by inserting into the original expression (2.2) a unity of the form $1 = \int \mathcal{D}\rho\delta(\rho - \rho_{\rm gas})$, exponentiating the δ-function upon the introduction of a Lagrange multiplier, and integrating then out all the fields except of the dynamical monopole density ρ. This procedure can be seen to be equivalent to the substitution

$$\exp\left[-\frac{1}{2}\int d^3x(\nabla\chi)^2\right] = \\ = \int\mathcal{D}\rho\exp\left[-\frac{g_m^2}{2}\int d^3xd^3y\rho(\mathbf{x})D_0(\mathbf{x}-\mathbf{y})\rho(\mathbf{y}) - ig_m\int d^3x\chi\rho\right] \tag{2.11}$$

into eq. (2.6) with the subsequent integration over the field χ, which then plays just the rôle of the Lagrange multiplier. In particular, the substitution (2.11) removes the kinetic term of the field χ, owing to which this integration can be performed in the saddle-point approximation. The respective saddle-point equation should be solved iteratively in g_m by setting $\chi = \chi^{(0)} + g_m\chi^{(1)}$. This eventually yields the following expression for the partition function in terms of the integral over monopole densities:

$$\mathcal{Z} = \int\mathcal{D}\rho\exp\left\{-\left[\frac{g_m^2}{2}\int d^3xd^3y\rho(\mathbf{x})D_0(\mathbf{x}-\mathbf{y})\rho(\mathbf{y}) + V[\rho]\right]\right\}. \tag{2.12}$$

[2]In this discussion, the difference between m_D and M is unimportant.

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The monopole potential $V[\rho]$ here reads

$$V[\rho] = \int d^3x \left[\rho \,\mathrm{arcsinh}\, \varrho - 2\zeta\sqrt{1+\varrho^2}\right] - \\ - 2(g_m\zeta)^2 \int d^3x d^3y \sqrt{1+\varrho^2(\mathbf{x})} D_m(\mathbf{x}-\mathbf{y}) \sqrt{1+\varrho^2(\mathbf{y})}\,, \qquad (2.13)$$

where $\varrho \equiv \rho/(2\zeta)$. The last term on the r.h.s. of this equation is again a leading (m_D/m)-correction to the respective expression in the limit when m is sent to infinity. In the dilute gas approximation, $|\rho| \ll \zeta$, eq. (2.13) becomes a simple quadratic functional:

$$V[\rho] \to \frac{1}{2}\left[\frac{1}{2\zeta} - \left(\frac{g_m}{m}\right)^2\right] \int d^3x \rho^2 \simeq \frac{g_m^2}{2M^2} \int d^3x \rho^2\,,$$

where the last equality is implied within the leading (m_D/m)-approximation adapted. This leads to the simple expression for the generating functional of correlators of the monopole densities within these two approximations:

$$\mathcal{Z}[j] \equiv \int \mathcal{D}\rho \exp\left\{-\left[\frac{g_m^2}{2}\int d^3x d^3y \rho(\mathbf{x}) D_0(\mathbf{x}-\mathbf{y})\rho(\mathbf{y}) + \frac{g_m^2}{2M^2}\int d^3x \rho^2 + \int d^3x j\rho\right]\right\} \\ = \exp\left[-\frac{M^2}{2g_m^2}\int d^3x d^3y j(\mathbf{x}) j(\mathbf{y}) \partial^2 D_M(\mathbf{x}-\mathbf{y})\right]. \qquad (2.14)$$

(Sending for a while m_D to zero (since it is exponentially small), we get from eq. (2.14): $\langle\rho(\mathbf{x})\rho(0)\rangle = 2\zeta\delta(\mathbf{x})$. This means that with the exponential accuracy the average density of monopoles is equal to 2ζ, which can also be seen directly from the $(|\rho| \ll \zeta)$-limit of eq. (2.13).) In particular, the Wilson loop reads:

$$\langle W(\mathcal{C})\rangle = \langle W(\mathcal{C})\rangle_{\rm free}\, \mathcal{Z}[i\eta] \qquad (2.15) \\ = \exp\left\{-\frac{g^2}{2}\left[\frac{M^2}{2}\int_\Sigma d\sigma_{\mu\nu}(\mathbf{x})\int_\Sigma d\sigma_{\mu\nu}(\mathbf{y}) + \oint_{\mathcal{C}} dx_\mu \oint_{\mathcal{C}} dy_\mu\right] D_M(\mathbf{x}-\mathbf{y})\right\}.$$

This equation can straightforwardly be derived by making use of the formula

$$\partial_\mu \eta(\mathbf{x}) = 2\pi\varepsilon_{\mu\nu\lambda}\left[2\partial_\nu^x \oint_{\mathcal{C}} dy_\lambda D_0(\mathbf{x}-\mathbf{y}) - \int_\Sigma d\sigma_{\nu\lambda}(\mathbf{y})\delta(\mathbf{x}-\mathbf{y})\right].$$

Here, $\eta(\mathbf{x}) = 2\pi\varepsilon_{\mu\nu\lambda}\partial_\mu^x \int_\Sigma d\sigma_{\nu\lambda}(\mathbf{y}) D_0(\mathbf{x}-\mathbf{y})$ is the solid angle under which an arbitrary surface Σ spanned by the contour $\mathcal{C}$ shows up to the observer located at the point $\mathbf{x}$. Also, in eq. (2.15), $\langle W(\mathcal{C})\rangle_{\rm free} = \exp\left[-\frac{g^2}{2}\oint_{\mathcal{C}} dx_\mu \oint_{\mathcal{C}} dy_\mu D_0(\mathbf{x}-\mathbf{y})\right]$ is the contribution of the free photons to the Wilson loop. The explicit Σ-dependence of the r.h.s. of eq. (2.15) appearing in the dilute gas approximation becomes eliminated by the summation over branches of the arcsinh-function in the full monopole action (2.12)–(2.13). This is the main principle of correspondence between fields and strings in compact QED proposed in ref. [2] in the language of the Kalb-Ramond field $h_{\mu\nu}$, $\varepsilon_{\mu\nu\lambda}\partial_\mu h_{\nu\lambda} \propto \rho$.

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As far as the string tension and the inverse coupling constant of the rigidity term [9] are concerned, those can be evaluated upon the derivative expansion of the Σ-dependent part of eq. (2.15). By virtue of the general formulae from ref. [10] we obtain

$$\sigma = 4\pi g^2 M \qquad \text{and} \qquad \alpha^{-1} = -\frac{\pi g^2}{2M}, \tag{2.16}$$

respectively. Clearly, both of these quantities represent the modifications of the standard ones, corresponding to the limit when m is considered to be infinitely large w.r.t. m_D. The standard expressions follow from eq. (2.16) upon the substitution into this equation $M \to m_D$.

3. RG flow at finite temperature

At non-zero temperature, the path integral (2.4) is defined with periodic boundary conditions in the interval $0 < t < \beta \equiv T^{-1}$. First straightforward estimate concerns the critical temperature of the Berezinskii-Kosterlitz-Thouless phase transition from the plasma phase to the phase where monopoles and antimonopoles are bound into molecules. Denoting by x the spatial 2D-vector and taking into account that at $|\mathsf{x}| \gg \beta$, $\frac{1}{|\mathsf{x}|} \to -2T\ln(\mu|\mathsf{x}|)$, $\frac{e^{-m|\mathsf{x}|}}{|\mathsf{x}|} \to 2TK_0(m|\mathsf{x}|)$, where μ stands for the IR cutoff, and K_0 denotes the modified Bessel function, we get for the mean squared separation between a monopole and an antimonopole at high temperatures:

$$\begin{aligned}\left\langle r^2\right\rangle &\sim \int d^2\mathsf{x}|\mathsf{x}|^{2-\frac{8\pi T}{g^2}} \exp\left[\frac{4\pi T}{g^2}K_0(m|\mathsf{x}|)\right] \\ &\sim \int^{\infty} d|\mathsf{x}||\mathsf{x}|^{3-\frac{8\pi T}{g^2}} \exp\left[\frac{(2\pi)^{3/2}T}{g^2(m|\mathsf{x}|)^{1/2}}e^{-m|\mathsf{x}|}\right].\end{aligned} \tag{3.1}$$

Clearly, the value of the critical temperature [8] $T_c = g^2/(2\pi)$ (above which the integral converges, which signals on the appearance of the monopole-antimonopole molecules) is not affected by the Higgs-motivated exponential factor. That is because at arbitrary temperature, the latter one tends rapidly to unity at $|\mathsf{x}| \to \infty$.

Let us now proceed with the RG analysis of the leading (m_D/m)-part of the partition function (2.6), which has the form

$$\mathcal{Z} = \int \mathcal{D}\chi \exp\left\{\int d^3x \left[\frac{1}{2}\chi\left(\partial_x^2 + \partial_y^2 + \partial_t^2\right)\chi + 2\zeta\cos(g_m\chi) + \left(\frac{g_m\zeta}{m}\right)^2\cos(2g_m\chi)\right]\right\}. \tag{3.2}$$

In this way, we shall apply the techniques elaborated out in refs. [11, 12] for the case when m is considered to be infinitely large w.r.t. m_D. First, let us split the cutoff sine-Gordon field as $\chi_\Lambda = \chi_{\Lambda'} + h$, where the field h includes the modes with the momenta lying in the range between Λ' and Λ and with all possible Matsubara

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frequencies. Then, integrating the h-field out and denoting $f(\mathbf{x}) \equiv 1+(g_m/m)^2\delta(\mathbf{x})$, one gets

$$\mathcal{Z} = \int \mathcal{D}\chi_{\Lambda'} \exp\left[-\frac{1}{2}\int d^3x\, (\nabla\chi_{\Lambda'})^2\right] \times$$
$$\times \exp\Bigg\{2\zeta \int d^3x \left\langle \cos\left(g_m\chi_\Lambda\right)\right\rangle_h +$$
$$+ 2\zeta^2 \int d^3x d^3y \Big[\left\langle \cos\left(g_m\chi_\Lambda(\mathbf{x})\right)\cos\left(g_m\chi_\Lambda(\mathbf{y})\right)\right\rangle_h f(\mathbf{x}-\mathbf{y}) -$$
$$- \left\langle \cos\left(g_m\chi_\Lambda(\mathbf{x})\right)\right\rangle_h \left\langle \cos\left(g_m\chi_\Lambda(\mathbf{y})\right)\right\rangle_h\Big]\Bigg\},$$

where $\langle\ldots\rangle_h$ means the average w.r.t. the action $\frac{1}{2}\int d^3x(\nabla h)^2$. By calculating the averages, the argument of the last exponent can be written as

$$2\zeta A(0) \int d^3x \cos\left(g_m\chi_{\Lambda'}\right) + (\zeta A(0))^2 \times \tag{3.3}$$
$$\times \sum_{k=0,1} \int d^3x d^3y \left[A^{2(-)^k}(\mathbf{x}-\mathbf{y}) f(\mathbf{x}-\mathbf{y}) - 1\right] \cos\left[g_m\left(\chi_{\Lambda'}(\mathbf{x}) + (-)^k\chi_{\Lambda'}(\mathbf{y})\right)\right],$$

where $A(\mathbf{x}) = \exp(-\frac{g_m^2}{2}G(\mathbf{x}))$ with $G(\mathbf{x})$ denoting the propagator of the field h. (Clearly, G is different from D_0, since the momenta of the field h vary only in the finite interval (Λ',Λ).) Similarly to ref. [12], we shall investigate the generalized model where $G(0)$ is considered to be finite. Next, one can pass to the integration over center-of-mass and relative-distance coordinates, $\mathbf{z} = (\mathbf{x}+\mathbf{y})/2$ and $\mathbf{u} = \mathbf{x}-\mathbf{y}$, and taking into account that $A^{\pm 2}(\mathbf{u}) \to 1$ at $|\mathbf{u}| \to \infty$, Taylor expand in $\mathbf{u}$ the last integrand on the r.h.s. of eq. (3.3). This yields for the respective integral the following expression:

$$\int d^3z \left[\chi_{\Lambda'}(\mathbf{z})\left(\sum_\mu \eta_\mu^2\partial_\mu^2\right)\chi_{\Lambda'}(\mathbf{z}) + \left(\frac{g_m A(0)}{m}\right)^2 \cos\left(2g_m\chi_{\Lambda'}(\mathbf{z})\right)\right],$$

where $\eta_\mu = \frac{g_m^2}{2}\int d^3u u_\mu^2[A^{-2}(\mathbf{u})f(\mathbf{u}) - 1]$. Introducing the notation $\zeta_\mu^{-2} = 1 + 2\zeta^2 A^2(0)\eta_\mu$, rescaling the field $\chi_{\Lambda'}$ as $\chi = \zeta_x^{-1}\chi_{\Lambda'}$ and momenta as $\mathbf{p}' = (\Lambda/\Lambda')\mathbf{p}$, and denoting $\gamma = (\frac{\Lambda}{\Lambda'}\frac{\zeta_x}{\zeta_t})^2$, we eventually get the following expression for the partition function (3.2):

$$\mathcal{Z} = \int \mathcal{D}\chi \exp\Bigg\{\int d^3x \Bigg[\frac{1}{2}\chi\left(\partial_x^2 + \partial_y^2 + \gamma\partial_t^2\right)\chi + \tag{3.4}$$
$$+ 2\zeta A(0)\cos(g_m\zeta_x\chi) + \left(\frac{g_m\zeta A^2(0)}{m}\right)^2 \cos(2g_m\zeta_x\chi)\Bigg]\Bigg\}.$$

To derive the RG equations, one should consider eq. (3.4) at the infinitesimal transformation $\Lambda' = \Lambda - \delta\Lambda$ and compare it with the original expression (3.2). Clearly,

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this comparison leads to the RG equation describing the evolution of m, which is new w.r.t. the standard equations describing the evolution of ζ and g_m. In this way, the following formulae, which can be straightforwardly derived, are useful:

$$\frac{d\zeta_x}{d\Lambda} = \frac{g_m^4 A^2(0)\mathcal{J}_2\zeta^2}{4\pi\Lambda^5}\,, \qquad \frac{dA(0)}{d\Lambda} = \frac{\pi x}{\Lambda}\frac{\tau}{2}\coth\frac{\tau}{2}\,.$$

Here, $d\Lambda \equiv -\delta\Lambda$, $x = Tg_m^2/(4\pi^2)$, $\tau = \Lambda/(T\sqrt{\gamma})$, and (within the notations of ref. [12]) $\mathcal{J}_2 \equiv \pi\int_0^\infty d\xi\xi^3 J_0(\xi)$ with J_0 standing for the Bessel function. Following ref. [12], one can introduce one more dimensionless parameter $z = (2\pi)^3\mathcal{J}_2 A^2(0)\frac{\zeta}{\Lambda^2 T}$ and by denoting $t = -\ln(\Lambda/T)$ get the following system of RG equations:

$$dx = -x^3z^2dt\,, \qquad dz^2 = -2z^2\left(\pi x\frac{\tau}{2}\coth\frac{\tau}{2} - 2\right)dt\,, \qquad d\ln\tau = -2dt\,. \tag{3.5}$$

Next, we can also introduce a novel, m-dependent, dimensionless parameter

$$u \equiv \Lambda^{-3/4}\sqrt{g_m\zeta m^{-1}}$$

and comparing eqs. (3.4) and (3.2) derive for it the following RG equation:

$$d\ln u = \left(\frac{3}{4} - \pi x\frac{\tau}{2}\coth\frac{\tau}{2}\right)dt\,. \tag{3.6}$$

This is just the announced new equation which describes indirectly the evolution of m.

In the limit $t \to \infty$ (or $\tau \to 0$), the RG flow (3.5) in the model under study becomes that of the 2D XY model. In particular, the BKT transition point of the XY model, $z_c = 0$, $x_c = 2/\pi$, can straightforwardly be read off from eq. (3.5). It is worth noting that the critical value x_c corresponds to the critical temperature $T_c = g^2/(2\pi)$, which was obtained above heuristically, without any RG analysis. In the vicinity of the BKT transition point, the RG trajectories stemming from the integration of the first two of eqs. (3.5) are typical hyperbolae of the XY-model. They are defined by the equation $(x - x_c)^2 - (2/\pi)^3z^2 = C_1$, where C_1 stands for some constant.

In the same XY-model limit, the flow of u takes the form

$$d\ln u = \left(\frac{3}{4} - \pi x\right)dt\,. \tag{3.7}$$

This equation yields the flow of m itself: $m = Cg_m\zeta T^{-3/2}(T/\Lambda)^{Tg_m^2/(2\pi)}$, where C is g_m-independent constant of integration. The limit (2.5) is then satisfied provided that the following inequality holds: $g_m^3(T/\Lambda)^{Tg_m^2/(2\pi)} \ll T^{3/2}/(C\zeta)$. Owing to eq. (2.3), it can be rewritten as $g_m\eta^7(T/\Lambda)^{Tg_m^2/\pi} \ll C^{-2}T^3$. We see that in the limit of small Λ under discussion, this inequality is indeed satisfied for any given C at sufficiently small g_m. This means that in the XY-model limit, the RG flow does not lead us

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outside the scope of the original approximation (2.5). Finally, in the vicinity of the BKT transition point, eq. (3.7) can be integrated together with the first two of eqs. (3.5). This yields $u = \exp[-5(x - x_c)/(4C_1) + C_2]$, where C_2 is a certain constant independent of C_1.

4. String representation of the Wilson loop in the SU(3)-case

In the SU(3)-generalization of the Georgi-Glashow model, the monopole density $\rho_{\rm gas}$ entering eq. (2.2) should be replaced by the following one [13, 14]: $\vec\rho_{\rm gas}(\mathbf{x}) = \sum_a \vec q_{i_a}\delta(\mathbf{x} - \mathbf{z}_a)$. Here, $\vec q_i$'s are the root vectors of the group SU(3):

$$\vec q_1 = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right), \qquad \vec q_2 = (-1, 0), \qquad \vec q_3 = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right), \qquad \vec q_{-i} = -\vec q_i\,.$$

The partition function (2.4) becomes modified to

$$\mathcal{Z} = \int \mathcal{D}\vec\chi\mathcal{D}\psi \exp\left\{-\int d^3x \left[\frac{1}{2}(\nabla\vec\chi)^2 + \frac{1}{2}(\nabla\psi)^2 + \frac{m^2}{2}\psi^2 - \right.\right.$$
$$\left.\left. - 2\zeta e^{g_m\psi}\sum_{i=1}^{3}\cos\left(g_m\vec q_i\vec\chi\right)\right]\right\},$$

where $\vec\chi = (\chi^1, \chi^2)$ and analogously to ref. [13] we have assumed that the W-bosons corresponding to different root vectors have the same masses.

Similarly to section 2, in the limit (2.5), the field ψ can be integrated out, and we get the following SU(3)-version of eq. (2.6):

$$\mathcal{Z} = \int \mathcal{D}\vec\chi \exp\left\{-\int d^3x\left[\frac{1}{2}(\nabla\vec\chi)^2 - 2\zeta\sum_{i=1}^{3}\cos\left(g_m\vec q_i\vec\chi\right)\right] + \right. \tag{4.1}$$
$$\left. +2(g_m\zeta)^2\sum_{i,j=1}^{3}\int d^3x d^3y \cos\left(g_m\vec q_i\vec\chi(\mathbf{x})\right) D_m(\mathbf{x}-\mathbf{y})\cos\left(g_m\vec q_j\vec\chi(\mathbf{y})\right)\right\}.$$

The full Debye mass stemming from the expansion of cosines in this equation can straightforwardly be obtained by noting that $\sum_{i=1}^{3}(\vec q_i\vec\chi)^2 = \frac{3}{2}\vec\chi^{\,2}$. It reads $M^2 = m_D^2(1 + 2m_D^2/m^2)$, where $m_D^2 = 3g_m^2\zeta$ is the squared standard Debye mass in the limit when m is sent to infinity.

In order to derive the potential of monopole densities, one should again perform the substitution (2.11) with the replacements $\chi \to \vec\chi$, $\rho \to \vec\rho$. The resulting saddle-point equation,

$$\sum_{i=1}^{3}\vec q_i \sin(g_m\vec q_i\vec\chi(\mathbf{x}))\left[1 + 2g_m^2\zeta\sum_{j=1}^{3}\int d^3y D_m(\mathbf{x}-\mathbf{y})\cos(g_m\vec q_j\vec\chi(\mathbf{y}))\right] = -\frac{i}{2\zeta}\vec\rho(\mathbf{x})\,,$$

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should again be solved iteratively by setting $\vec{\chi} = \vec{\chi}^{(0)} + g_m \vec{\chi}^{(1)}$. This equation can be solved w.r.t. $\vec{q}_i \vec{\chi}$ by representing $\vec{\rho}$ as [15] $\sum_{i=1}^{3} \vec{q}_i \rho_i$, where $\rho_1 \equiv \left(\rho^1/\sqrt{3} + \rho^2\right)/\sqrt{3}$, $\rho_2 \equiv -2\rho^1/3$, $\rho_3 \equiv \left(\rho^1/\sqrt{3} - \rho^2\right)/\sqrt{3}$. The so-obtained solution has the form

$$\begin{aligned}
\vec{q}_i \vec{\chi}^{(0)}(\mathbf{x}) &= -\frac{i}{g_m} \operatorname{arcsinh} \varrho_i(\mathbf{x}) \,, \\
\vec{q}_i \vec{\chi}^{(1)}(\mathbf{x}) &= \frac{i\rho_i(\mathbf{x})}{\sqrt{1+\varrho_i^2(\mathbf{x})}} \sum_{j=1}^{3} \int d^3y D_m(\mathbf{x}-\mathbf{y}) \sqrt{1+\varrho_j^2(\mathbf{y})} \,,
\end{aligned}$$

where $\varrho_i \equiv \rho_i/(2\zeta)$. This yields the desired representation of the partition function in the form of eq. (2.12) with ρ replaced by $\vec{\rho}$ and $V[\rho]$ replaced by

$$\begin{aligned}
V[\vec{\rho}] = &\int d^3x \sum_{i=1}^{3} \left[\rho_i \operatorname{arcsinh} \varrho_i - 2\zeta\sqrt{1+\varrho_i^2}\right] - \\
&- 2(g_m\zeta)^2 \int d^3x d^3y \sum_{i,j=1}^{3} \sqrt{1+\varrho_i^2(\mathbf{x})} D_m(\mathbf{x}-\mathbf{y}) \sqrt{1+\varrho_j^2(\mathbf{y})} \,.
\end{aligned}$$

Similarly to the SU(2)-case, in the dulite gas approximation, $|\vec{\rho}| \ll \zeta$, the monopole potential becomes a quadratic functional:

$$V[\vec{\rho}] \to \frac{1}{3}\left(\frac{1}{2\zeta} - \frac{3g_m^2}{m^2}\right) \int d^3x \vec{\rho}^{\,2} \simeq \frac{g_m^2}{2M^2} \int d^3x \vec{\rho}^{\,2} \,,$$

where the last equality is again implied within the leading (m_D/m)-approximation.

The monopole part of the Wilson loop can be written as

$$\langle W(\mathcal{C})\rangle_{\rm mon} = \frac{1}{3} \sum_{\alpha=1}^{3} \left\langle \exp\left(i\int d^3x \vec{\rho}\vec{\mu}_\alpha \eta\right)\right\rangle . \tag{4.2}$$

Here, $\vec{\mu}_\alpha$'s are the weight vectors of the group SU(3), which are just the charges of a quark of the α-th colour w.r.t. the diagonal gluons: $\vec{\mu}_1 = (-1/2, 1/(2\sqrt{3}))$, $\vec{\mu}_2 = (1/2, 1/(2\sqrt{3}))$, $\vec{\mu}_3 = (0, -1/(\sqrt{3}))$. Note that in a derivation of eq. (4.2) we have used the identity $\operatorname{tr}\exp(i\vec{a}\frac{\vec{\lambda}}{2}) = \sum_{\alpha=1}^{3} \exp\left(i\vec{a}\vec{\mu}_\alpha\right)$ valid for an arbitrary vector $\vec{a}$. Also, in eq. (4.2), the average is performed w.r.t. the just discussed above partition function expressed in terms of $\vec{\rho}$'s.

In the dilute gas approximation, this partition function thus takes the form

$$\mathcal{Z} = \int \mathcal{D}\vec{\rho} \exp\left\{-\left[\frac{g_m^2}{2}\int d^3x d^3y \vec{\rho}(\mathbf{x}) D_0(\mathbf{x}-\mathbf{y}) \vec{\rho}(\mathbf{y}) + \frac{g_m^2}{2M^2}\int d^3x \vec{\rho}^{\,2}\right]\right\}.$$

The resulting gaussian integration can straightforwardly be performed and yields

$$\begin{aligned}
\langle W(\mathcal{C})\rangle_{\rm mon} = \exp\Bigg\{-\frac{g^2}{6}\Bigg[&\frac{M^2}{2}\int_\Sigma d\sigma_{\mu\nu}(\mathbf{x}) \int_\Sigma d\sigma_{\mu\nu}(\mathbf{y}) D_M(\mathbf{x}-\mathbf{y}) + \\
&+ \oint_{\mathcal{C}} dx_\mu \oint_{\mathcal{C}} dy_\mu \left(D_M(\mathbf{x}-\mathbf{y}) - D_0(\mathbf{x}-\mathbf{y})\right)\Bigg]\Bigg\} ,
\end{aligned}$$

where it has been taken into account that for any α, $\vec{\mu}_\alpha^2 = 1/3$.

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An expression for the full Wilson loop can be obtained upon the multiplication of this result by the contribution brought about by the free photons. The latter one reads

$$\left\langle W(\mathcal{C})\right\rangle_{\mathrm{free}}=\frac{1}{3}\sum_{\alpha=1}^{3}\left\langle\exp\left(i\vec{\mu}_{\alpha}\oint_{\mathcal{C}}dx_{\mu}\vec{A}_{\mu}\right)\right\rangle=\exp\left(-\frac{g^{2}}{6}\oint_{\mathcal{C}}dx_{\mu}\oint_{\mathcal{C}}dy_{\mu}D_{0}(\mathbf{x}-\mathbf{y})\right),$$

where $\langle\ldots\rangle$ stands for the average w.r.t. the action $\frac{1}{4g^2}\int d^3x\vec{F}_{\mu\nu}^2$. The full Wilson loop then has the form of the r.h.s. of eq. (2.15) with g^2 replaced by $g^2/3$. Clearly, the SU(3)-expressions for σ and α^{-1} can be obtained from eq. (2.16) by performing in that equation the same replacement.

5. Conclusions

In the present paper, we have explored confining strings and RG flow in the (2+1)-dimensional Georgi-Glashow model. This has been done in a certain limit (2.5), (2.8) of this model and in the leading (m_D/m)-approximation, where m_D is the Debye mass of the dual photon and m is the mass of the Higgs boson. Both in the SU(2)- and SU(3)-cases, we have derived the respective Higgs-inspired corrections to m_D, to the potential of monopole densities, and as a consequence, to the string tension and rigidity coupling constant of the confining string. As far as the RG analysis is concerned, for the SU(2)-theory there has been obtained a novel equation describing the evolution of the Higgs mass. This equation takes a remarkably simple form in the limit when the original theory goes over to the 2D XY model. Owing to this fact, it has been checked that the evolution of the Higgs mass in this limit does not violate the adapted approximation (2.5).

In the present paper, we have disregarded possible effects brought about by the W-bosons. Recently [16, 17], the influence of these bosons to the dynamics of the phase transition has been studied by treating them as vortices of a certain disorder operator. In particular for SU(N)-case, the respective RG-analysis has been performed in ref. [17]. In the forthcoming paper [18], we plan to take into account the effects of W-bosons to the theory of confining strings at finite temperature.

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